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AUXILIARY INFORMATION BASED GENERALLY WEIGHTED MOVING AVERAGE FOR PROCESS MEAN

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Abstract: The univariate mean process monitoring only used the information from the study variable. One of the univariate control chart that used to monitor the mean process is GWMA control chart. But, in this research, we needed to monitor process mean used the information from the study variable and information on the adding or auxiliary variable. The enhanced control chart in this research named AIB-GWMA control chart. In this research, we also made a comparison between the GWMA and AIB-GWMA to know the sensitivity and effectiveness of these control chart. The comparison used to know the effect of the auxiliary variable in process monitoring. The performance of these control chart evaluated using Average Run Length with help of Monte Carlo simulation. The result of this study was AIB-GWMA had a smaller ARL than the GWMA control chart. It showed that AIB-GWMA was faster to detect a shift in mean process. In further study, we recommended to enhance the performance of the AIB-GWMA by extending the current work to the AIB-MaxGWMA, so it is possible to monitor process mean and variance simultaneously.

1. INTRODUCTION

The main goal of statistical quality control was to monitor the production process and detect shift in the parameter of a process rapidly. Monitoring the production process used a statistical tool, that was control chart. Beside that, statistical quality control used in the production process to improve the output quality by reduced the variation in the production process [1]. Shewhart introduced the concept to made a control chart. To monitor the process mean, the control chart that can be used such as Cumulative Sum (CUSUM) developed by [2], Exponentially Weighted Moving Average (EWMA) by [3], and Generally Weighted Moving Average (GWMA) by [4]. These control chart had a good performance to detect a small shift in the process mean. For more recent research on the improved control chart, we referred to [5] and [6].

Monitoring the process mean of a quality characteristic (variable) used univariate control chart. Monitoring the process mean used univariate control chart only used the information from the study variable. But, the recent researches, monitoring the process mean of a study variable can be used the information on the study variable and the auxiliary variable, where auxiliary variable was a variable that had a correlation with the study variable. Control chart that used auxiliary variable in monitoring tha process mean was developed by [9] called Auxiliary Information Based Shewhart (AIB-Shewhart). They prove

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that AIB-Shewhart had a better performance than Classical Shewhart control chart. A control chart that used a regression estimator proposed by [10] to monitor a mean process. This control chart showed a better performance than the Shewhart's $\bar{X}$-chart. MxEWMA control chart developed by [11] for process mean monitoring, where this control chart also based on the regression estimator. They made a comparison between MxEWMA and some recent CUSUM and EWMA control chart. The result of the research was MxEWMA had a better performance than some recent CUSUM and EWMA control chart. So many researches that used the auxiliary variable in monitoring process mean prove that monitoring the process mean of a study variable was better when used the information from the study variable and auxiliary variable than without used auxiliary variable because control chart that used auxiliary variable had the estimator with smaller variance. The other researches for the control chart that used auxiliary variable, we referred to [7], [8], [12], [13], [14].

Average Run Length (ARL) used to evaluate the performance of the control chart [15]. There are two type of ARL, that were in control and out of control ARL. If the process was in control, in control ARL will be large to avoid false alarm. If the process was out of control, out of control ARL will be small, it means the control chart was fast to detect a shift in the process. Monte Carlo simulation was the approach to compute ARL.

In this paper, we built a control chart to monitor the process mean that used auxiliary variable called AIB-GWMA control chart.

2. LITERATURE REVIEW

2.1 Generally Weighted Moving Average (GWMA) Control Chart

GWMA is defined as the moving average of a data set that involves the previous to the last subgroup.

If M is defined as the number of the subgroup until an event occurs counting from the occurance of the previous event, then the weights for each subgroup in the GWMA statistic are defined as follows:

$$P(M=r)=P(M=1)+P(M=2)+\ldots+P(M=j)+P(M>j)$$
$$P(M=r)=\left(\bar{P}_0-\bar{P}_1\right)+\left(\bar{P}_1-\bar{P}_2\right)+\ldots+\left(\bar{P}_{j-1}-\bar{P}_j\right)+\bar{P}_j \tag{1}$$

where

$$\sum_{r=1}^{\infty} P(M=r)=1$$

The weight for the last subgroup is $P(M=1)$, the weight for the previous subgroup is $P(M=2)$, and so on until $P(M=j)$ is the weight in the first subgroup.

If known Y_{ij} is an observation on Y variable. $\bar{Y}_j$ is the mean at the j-th subgroup. G_0 is the starting value, generally $G_0=\mu$. Then the GWMA statistics for process mean, G_j, can be written as follows:

$$G_j=P(M=1)\bar{Y}_j+P(M=2)\bar{Y}_{j-1}+\ldots+P(M=j)\bar{Y}_1+P(M>j)G_0$$
$$G_j=\left(\bar{P}_0-\bar{P}_1\right)\bar{Y}_j+\left(\bar{P}_1-\bar{P}_2\right)\bar{Y}_{j-1}+\ldots+\left(\bar{P}_{j-1}-\bar{P}_j\right)\bar{Y}_1+\bar{P}_j\mu \tag{2}$$

The expectation value of G_j as follows:

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$$E(G_j) = E\left(\left(\bar{P}_0 - \bar{P}_1\right)\bar{Y}_j + \left(\bar{P}_1 - \bar{P}_2\right)\bar{Y}_{j-1} + \ldots + \left(\bar{P}_{j-1} - \bar{P}_j\right)\bar{Y}_1 + \bar{P}_j\mu\right)$$
$$E(G_j) = \left(\left(\bar{P}_0 - \bar{P}_1\right) + \left(\bar{P}_1 - \bar{P}_2\right) + \ldots + \left(\bar{P}_{j-1} - \bar{P}_j\right)\right)E\left(\bar{Y}\right) + E\left(\bar{P}_j\mu\right)$$
$$E(G_j) = \mu \tag{3}$$

The variance of G_j as follows:

$$\operatorname{var}(G_j) = \operatorname{var}\left(\left(\bar{P}_0 - \bar{P}_1\right)\bar{Y}_j + \left(\bar{P}_1 - \bar{P}_2\right)\bar{Y}_{j-1} + \ldots + \left(\bar{P}_{j-1} - \bar{P}_j\right)\bar{Y}_1 + \bar{P}_j\mu\right)$$
$$\operatorname{var}(G_j) = \left(\left(\bar{P}_0 - \bar{P}_1\right)^2 + \left(\bar{P}_1 - \bar{P}_2\right)^2 + \ldots + \left(\bar{P}_{j-1} - \bar{P}_j\right)^2\right)\operatorname{var}\left(\bar{Y}\right) + \operatorname{var}\left(\bar{P}_j\mu\right)$$
$$\operatorname{var}(G_j) = \left(\left(\bar{P}_0 - \bar{P}_1\right)^2 + \left(\bar{P}_1 - \bar{P}_2\right)^2 + \ldots + \left(\bar{P}_{j-1} - \bar{P}_j\right)^2\right)\frac{\sigma^2}{n} \tag{4}$$

$\bar{P}_j$ replaced with q^{j^ω} to facilitate the computation process, where $q = 1 - \lambda$ is a constant that has a value between 0 and 1. ω is adjustment parameter determined by the researcher. Then the equation 2 can be written as follows:

$$G_j = \left(\bar{P}_0 - \bar{P}_1\right)\bar{Y}_j + \left(\bar{P}_1 - \bar{P}_2\right)\bar{Y}_{j-1} + \ldots + \left(\bar{P}_{j-1} - \bar{P}_j\right)\bar{Y}_1 + \bar{P}_j\mu$$
$$G_j = \left(q^{0^\omega} - q^{1^\omega}\right)\bar{Y}_j + \left(q^{1^\omega} - q^{2^\omega}\right)\bar{Y}_{j-1} + \ldots + \left(q^{(j-1)^\omega} - q^{j^\omega}\right)\bar{Y}_1 + q^{j^\omega}\mu \tag{5}$$

The expectation value of G_j in the equation 5 as follows:

$$E(G_j) = E\left(\left(q^{0^\omega} - q^{1^\omega}\right)\bar{Y}_j + \left(q^{1^\omega} - q^{2^\omega}\right)\bar{Y}_{j-1} + \ldots + \left(q^{(j-1)^\omega} - q^{j^\omega}\right)\bar{Y}_1 + q^{j^\omega}\mu\right)$$
$$E(G_j) = \left(\left(q^{0^\omega} - q^{1^\omega}\right) + \left(q^{1^\omega} - q^{2^\omega}\right) + \ldots + \left(q^{(j-1)^\omega} - q^{j^\omega}\right)\right)E\left(\bar{Y}\right) + E\left(q^{j^\omega}\mu\right)$$
$$E(G_j) = \mu \tag{6}$$

The variance of G_j in the equation 5 as follows:

$$\operatorname{var}(G_j) = \operatorname{var}\left(\left(q^{0^\omega} - q^{1^\omega}\right)\bar{Y}_j + \left(q^{1^\omega} - q^{2^\omega}\right)\bar{Y}_{j-1} + \ldots + \left(q^{(j-1)^\omega} - q^{j^\omega}\right)\bar{Y}_1 + q^{j^\omega}\mu\right)$$
$$\operatorname{var}(G_j) = \left(\left(q^{0^\omega} - q^{1^\omega}\right)^2 + \left(q^{1^\omega} - q^{2^\omega}\right)^2 + \ldots + \left(q^{(j-1)^\omega} - q^{j^\omega}\right)^2\right)\operatorname{var}\left(\bar{Y}\right) + \operatorname{var}\left(q^{j^\omega}\mu\right)$$
$$\operatorname{var}(G_j) = \left(\left(q^{0^\omega} - q^{1^\omega}\right)^2 + \left(q^{1^\omega} - q^{2^\omega}\right)^2 + \ldots + \left(q^{(j-1)^\omega} - q^{j^\omega}\right)^2\right)\frac{\sigma^2}{n} \tag{7}$$

Based on the expectation value in equation 6 and variance in equation 7, GWMA control chart have the control limit as follows:

$$UCL = E(G_j) + L\sqrt{\operatorname{var}(G_j)} \tag{8}$$

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$$CL = E\left(G_j\right) \tag{9}$$

$$LCL = E\left(G_j\right) - L\sqrt{\operatorname{var}\left(G_j\right)} \tag{10}$$

where L is the distance from the control limit to the center line, where L expressed in standard deviation unit. If the GWMA statistic exceed the UCL or less than LCL, so the process is out of control [4].

2.2 AIB-GWMA Control Chart

Auxiliary Information Based Generally Weghting Moving Average (AIB-GWMA) control chart is used to monitor the process mean of a study variable using the information from the study variable and auxiliary variable.

Let $\left(Y_i, X_i\right)$ is a random sample with size n, where $\left(Y_i, X_i\right)$ is Normally distributed. Y as study variable and X as auxiliary variable, where there is correlation between Y and X. The mean and variance of Y are μ_Y and σ_Y^2. The mean and variance of X are μ_X and and σ_X^2. ρ_{XY} is the correlation between Y and X.

Let $\left(Y_{ij}, X_{ij}\right)$ be a bivariate random sample where i indexes the sample number and j indexes the subgroup number. Mean for each subgroup for quality characteristic Y and X are as follows:

$$\bar{Y}_j = \frac{\sum_{i=1}^{n} Y_{ij}}{n} \text{ and } \bar{X}_j = \frac{\sum_{i=1}^{n} X_{ij}}{n} \tag{11}$$

The variance for each subgroup for quality characteristic Y and X are as follows:

$$S_{Y,j}^2 = \frac{\sum_{i=1}^{n}\left(Y_{ij} - \bar{Y}_j\right)^2}{n} \text{ and } S_{X,j}^2 = \frac{\sum_{i=1}^{n}\left(X_{ij} - \bar{X}_j\right)^2}{n} \tag{12}$$

If the process is in control, then the developed estimator for mean as follows:

$$D_{Y,j} = \bar{Y}_j + \rho_{XY}\left(\frac{\sigma_Y}{\sigma_X}\right)\left(\mu_X - \bar{X}_j\right) \tag{13}$$

where to estimate the ρ_{XY} is using the formula:

$$\hat{\rho}_{XY} = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{(n-1) S_X S_Y} = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sqrt{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 \sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}} \tag{14}$$

Then the estimator in the equation 13 is transformed into the following estimator:

$$A_{Y,j} = \frac{D_{Y,j} - \mu_Y}{\sigma_Y\sqrt{\frac{1-\rho_{XY}^2}{n}}} \tag{15}$$

By using $A_{Y,j}$ in the equation 15, AIB-GWMA statistic are formed as follows:

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$$A_{Y,j}^{(1)} = \left(q^{0^{\omega}} - q^{1^{\omega}}\right) A_{Y,j} + \left(q^{1^{\omega}} - q^{2^{\omega}}\right) A_{Y,j-1} + \ldots + \left(q^{(j-1)^{\omega}} - q^{j^{\omega}}\right) A_{Y,1} + q^{j^{\omega}} \mu \tag{16}$$

When the process said to be in control, $A_{Y,j}^{(1)}$ has the mean μ and the variance $Q = \left(\left(q^{0^{\omega}} - q^{1^{\omega}}\right)^2 + \left(q^{1^{\omega}} - q^{2^{\omega}}\right)^2 + \ldots + \left(q^{(j-1)^{\omega}} - q^{j^{\omega}}\right)^2 \right) \frac{\sigma^2}{n}$.

The control limit for AIB-GWMA control chart is

$$UCL = E\left(A_{Y,j}^{(1)}\right) + L\sqrt{\operatorname{var}\left(A_{Y,j}^{(1)}\right)} \tag{17}$$

$$CL = E\left(A_{Y,j}^{(1)}\right) \tag{18}$$

$$LCL = E\left(A_{Y,j}^{(1)}\right) - L\sqrt{\operatorname{var}\left(A_{Y,j}^{(1)}\right)} \tag{19}$$

If the AIB-GWMA statistic exceed the UCL or less than LCL, so the process is out of control.

3. METHODOLOGY

3.1 Data Source

In this study, we used the simulation data. To evaluate the performance of the control chart, we used the out control ARL. The approach to compute out of control ARL was Monte Carlo simulation with determined in control ARL was 137. We made a data simulation with the different level of shift in process mean, that is 0, 0.5, 1, and 1.5 with different level of correlation, that is 0, 0.5, and 0.9.

3.2 Flow Chart

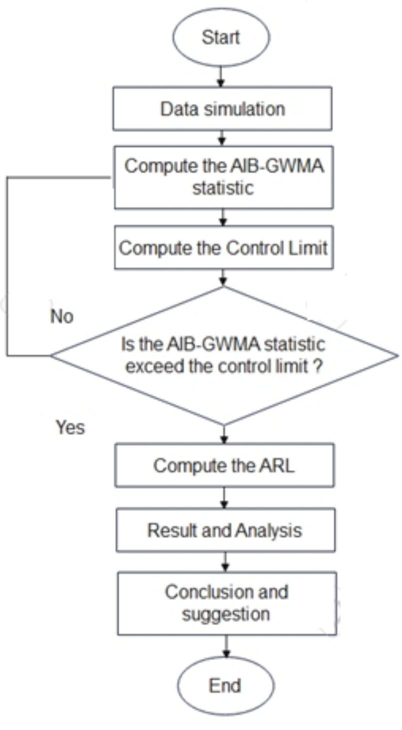

Figure 1. Flow Chart for the Research Methodology

4. RESULTS AND DISCUSSION

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In this study, we compared the GWMA and GWMA with the auxiliary variable (AIB-GWMA). We used out of control ARL to evaluated the performance of the control chart. When out of control ARL is smaller, it means that the control chart is faster to detect a shift in process mean.

We used Monte Carlo simulation to computed ARL with determined in control ARL is 137. We made a data simulation with the different level of shift in process mean, that is 0, 0.5, 1, and 1.5 with different level of correlation, that is 0, 0.5, and 0.9. In this study, we also used the different choice of q and ω. In Table 1 until 9 show the ARL for the GWMA and AIB-GWMA control chart for q = 0.5, 0.7, and 0.9 and ω= 0.5, 0.7, and 0.9 as follows

Table 1. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.5 and ω= 0.5

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	67.54	31.21	20.47	15.47
0.5	36.91	20.26	17.29	13.44
0.9	10.91	5.21	4.23	3.21

Table 2. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.7 and ω= 0.5

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	64.47	30.24	19.97	14.46
0.5	34.79	19.40	16.21	12.04
0.9	10.21	5.01	3.87	3.14

Table 3. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.9 and ω= 0.5

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	63.64	29.77	19.04	13.77
0.5	33.21	18.47	15.66	11.76
0.9	9.01	4.69	3.41	3.07

Table 4. The GWMA and AIB-GWMA ARL on different value of δ (mean shift)

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and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.5 and ω = 0.7

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	61.04	28.69	18.20	13.71
0.5	31.56	17.29	15.27	10.71
0.9	8.99	4.41	3.23	3.05

Table 5. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.7 and ω = 0.7

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	60.21	27.64	17.25	12.67
0.5	30.77	17.14	14.46	10.44
0.9	7.46	3.71	3.21	2.76

Table 6. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.9 and ω = 0.7

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	60.17	26.56	16.91	11.83
0.5	29.74	16.31	13.04	9.61
0.9	5.33	3.63	3.17	2.41

Table 7. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.5 and ω = 0.9

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	58.74	25.63	16.48	11.46
0.5	28.09	15.51	12.63	8.41
0.9	5.21	3.46	2.96	2.21

Table 8. The GWMA and AIB-GWMA ARL on different value of δ (mean shift) and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.7 and ω = 0.9

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	57.61	24.64	15.21	9.46
0.5	27.71	14.62	11.46	7.11
0.9	4.76	2.24	2.17	1.71

Table 9. The GWMA and AIB-GWMA ARL on different value of δ (mean shift)

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and ρ_{XY} (correlation between X and Y) with adjustment parameter q = 0.9 and ω = 0.9

ρ_{XY} (Level of correlation)	δ (Mean shift)			
	0	0.5	1	1.5
0	56.41	23.69	14.01	8.67
0.5	26.56	13.69	10.66	6.21
0.9	4.21	2.17	1.96	1.07

The data in Table 1 until 9 can be visualized in the Figure 2 until 10 as follows:

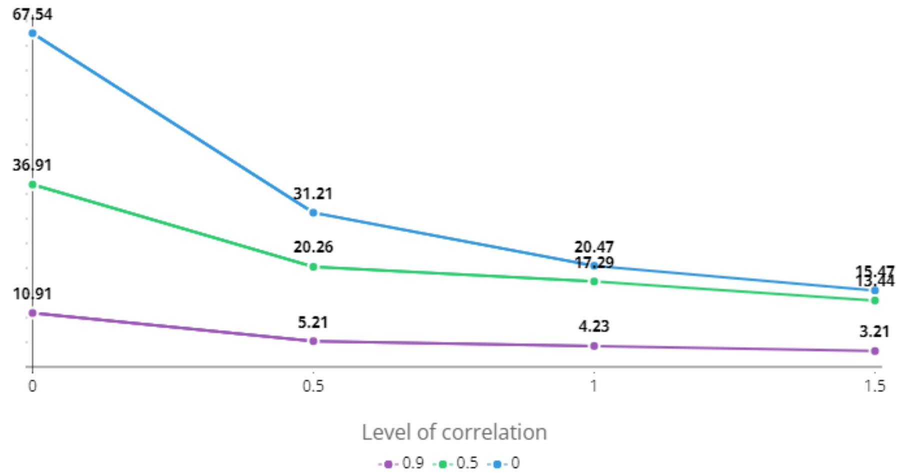

Figure 2. The ARL with q = 0.5 and ω = 0.5

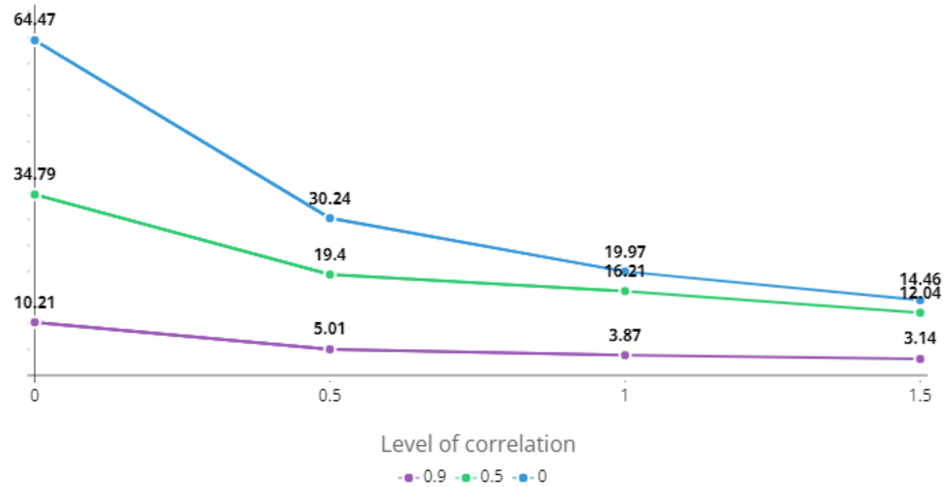

Figure 3. The ARL with q = 0.7 and ω = 0.5

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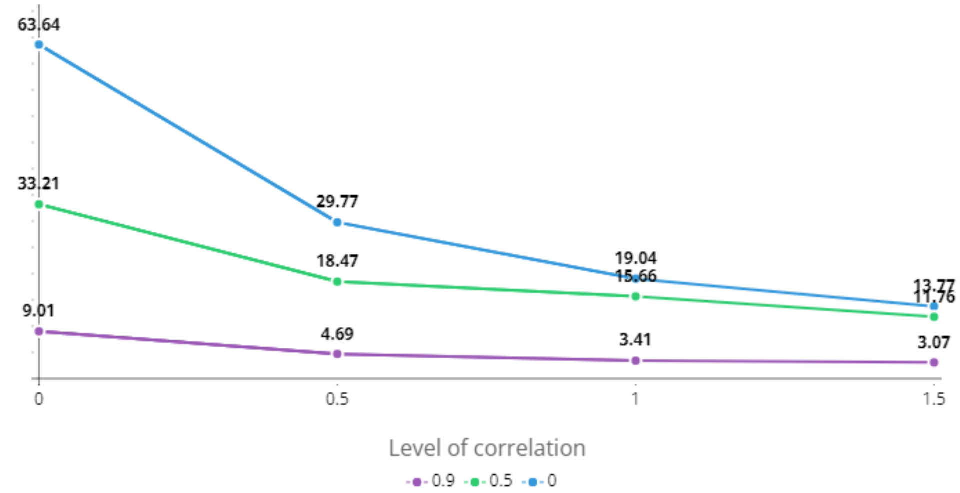

Figure 4. The ARL with q = 0.9 and ω = 0.5

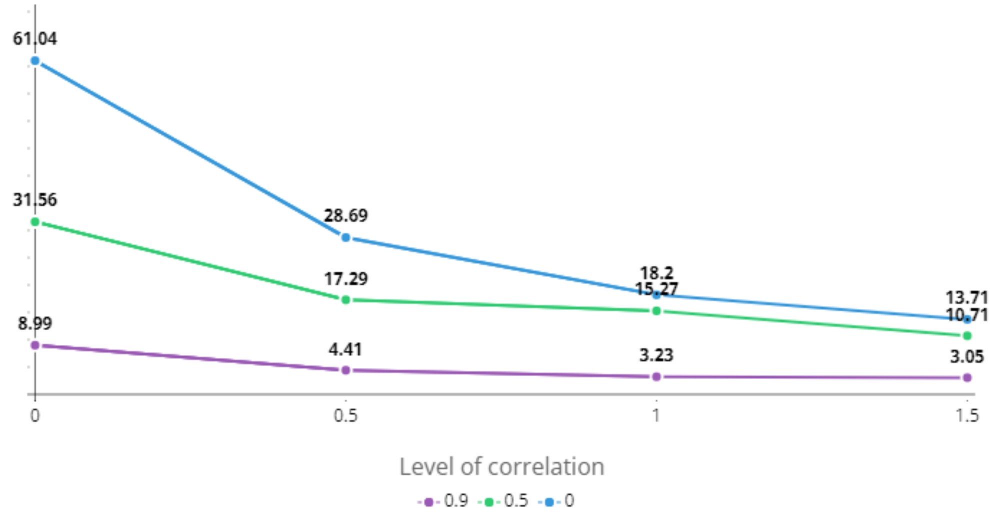

Figure 5. The ARL with q = 0.5 and ω = 0.7

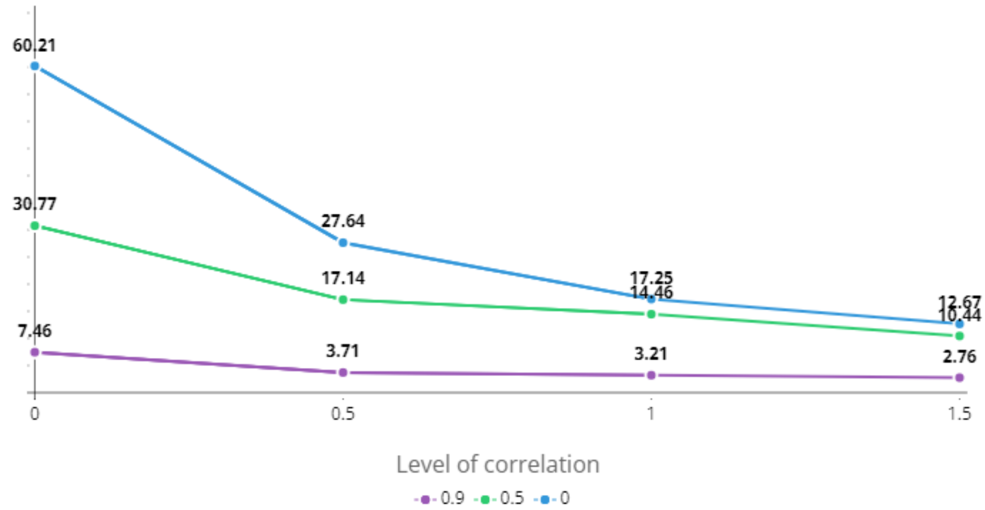

Figure 6. The ARL with q = 0.7 and ω = 0.7

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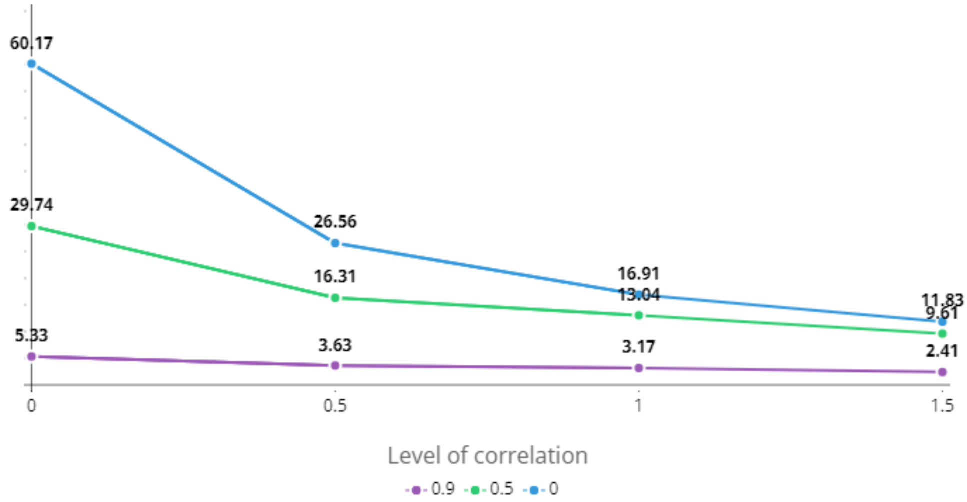

Figure 7. The ARL with q = 0.9 and ω = 0.7

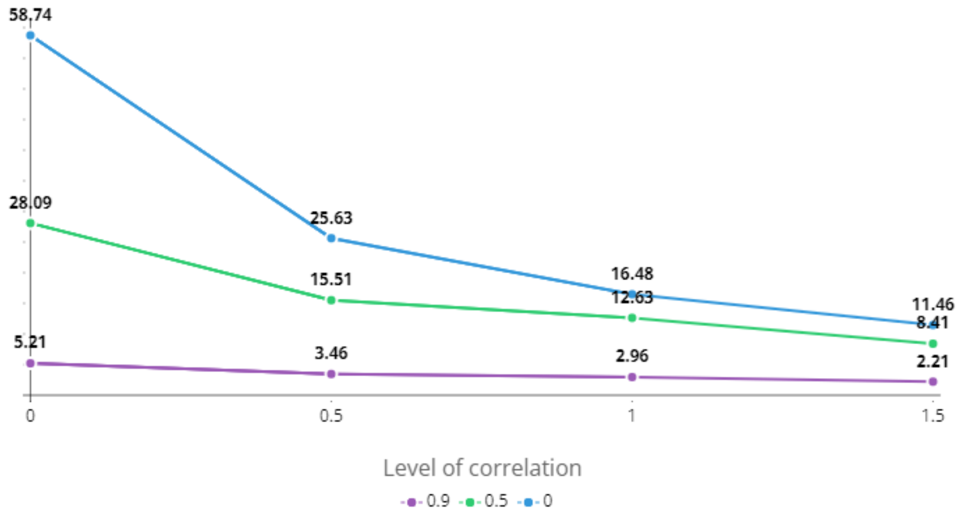

Figure 8. The ARL with q = 0.5 and ω = 0.9

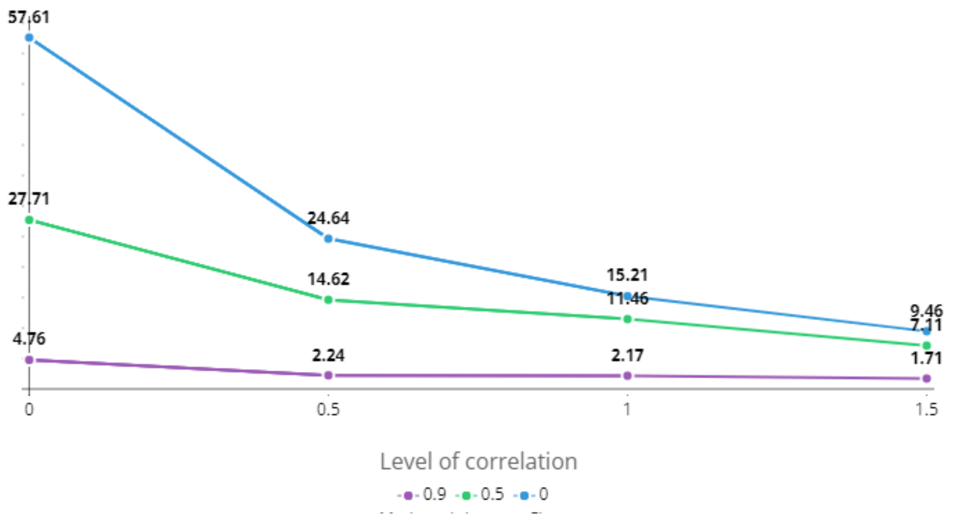

Figure 9. The ARL with q = 0.7 and ω = 0.9

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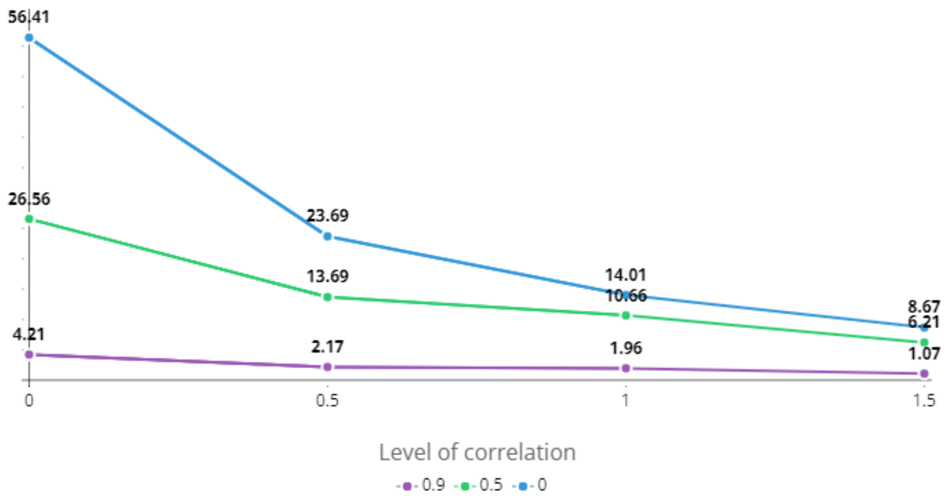

Figure 10. The ARL with q = 0.9 and ω = 0.9

From Figure 2 until 10, we can conclude that when the level of q and ω increase, the out of control ARL is smaller. It means that when the level of q and ω increase, the GWMA and AIB-GWMA control chart have a better performance. Beside that, when the level of correlation are increasing, the out of control ARL is getting smaller. It means that if the level of correlation increase, the control chart will be faster to detect shift in the process mean. It prove that AIB-GWMA control chart (with auxiliary variable) is better than the GWMA control chart (without auxiliary variable). The GWMA and AIB-GWMA will have same performance when there is no correlation between X and Y variable.

5. CONCLUSION

The conclusion of this study is we known that when including the auxiliary variable for monitoring the process mean is better than without using auxiliary variable. It can be seen in the figure 2 until 10 that when the level of correlation increase, the out of control ARL will getting smaller. The performance of AIB-GWMA will same with GWMA when there is no correlation. Beside that, as level of q and ω increase, the performance of GWMA and AIB-GWMA control chart will be better. We recommended to enhance the performance of the AIB-GWMA to the Auxiliary Information Based Maximum Generally Weighted Moving Average (AIB-MaxGWMA) that can used to monitor the process mean and variance simultaneously.

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